Technical Report No. 32-174

Applications for Monopropellants in Space Vehicles

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JET PROPULSION LABORATORY
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APPLICATIONS FOR MONOPROPELLANTS IN SPACE VEHICLES[*]

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ABSTRACT

The requirements of current and anticipated unmanned lunar and planetary spacecraft are discussed from the standpoint of the use of propulsion devices in guidance correction maneuvers. The requirements for post-injection maneuvers are reviewed specifically and the following is concluded: (1) early maneuvers--midcourse correction--have velocity increments of the order of 100 to 200 ft/sec for virtually all anticipated missions, whereas later maneuvers--approach correction--have velocity increments of the order of 200 to several hundred feet per second; (2) liquid-propellant systems offer significant weight advantages over solid-propellant systems where multiple operations are required; (3) minimum thrust level is determined by considerations of the guidance system; and (4) the gross weight of the midcourse propulsion system is normally 5% of the spacecraft or less.

The unique qualifications of monopropellants for these applications are discussed. As examples, the use of monopropellants in the Ranger and Mariner-A spacecraft is described, and possible future applications for monopropellants are outlined.

INTRODUCTION

Responsibility for the exploration of deep space through the use of unmanned spacecraft has been delegated to the Jet Propulsion Laboratory by the National Aeronautics and Space Administration. As a consequence, over the past few years a major part of the propulsion research, development, and systems analyses has been in the area of deep-space application. Such concentration of outlook has led to a fuller appreciation of the qualities which

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propellants and propulsion systems should possess for service in the space environment. For example, it is interesting to note that the temperature data telemetered from the Explorer satellites indicated that an internal temperature environment of $35 \pm 15°$ C was attained, leading to the conclusion that at least for certain missions, extended range of propellant serviceability is not particularly significant.

This paper represents the individual efforts of several contributors, each having developed a separate portion of the analyses to be described. It is the purpose of this paper to combine the facts and conclusions and thus to present a complete picture of the propulsion-system characteristics required for space-oriented applications.

DISCUSSION

In general, the discussion will be confined to post-injection guidance maneuvers. These maneuvers include both midcourse and approach correction maneuvers. A midcourse maneuver is a single impulse or perhaps a series of small impulses made relatively early in flight to eliminate errors in the injection guidance system and the boost propulsion performance. However, even if the desired trajectory is thereby achieved, it may prove to be incorrect because of uncertainties in the astronomical measurements. There is the additional possibility of introducing new guidance or propulsion errors while the midcourse maneuver is made. Therefore, another correction maneuver, known as the approach correction, is envisioned for many of the missions. It is expected that this maneuver would be made relatively late in flight and would serve to place the spacecraft in the correct position to undertake the terminal maneuver.

It appears that guidance and propulsion technology will not be sufficiently advanced, at least in the foreseeable future, to preclude the necessity for such corrections. Furthermore, the utmost precision will continue to be required. For example, an error of 1 ft/sec in velocity on a lunar mission would result in a miss distance of approximately 50 miles. When it is recalled that the injection velocity for the lunar mission is in excess of 30,000 ft/sec, this points out the extreme degree of precision which is needed for vehicle-borne guidance systems. If it is then concluded that some form of midcourse and approach correction is inevitable, the requirement for high precision in the on-board guidance equipment may be considerably relaxed, since Earth-bound computers can be used to calculate these corrections. The small amount of additional weight required by the propulsion system is more than compensated for by savings in guidance-system weight and complexity.

Lunar and Planetary Exploration Programs. With the need for post-injection maneuvers now established, it is appropriate to consider the propulsion system requirements in relation to the national unmanned-spaceflight effort. The flight program has been relatively well defined. The Apollo project undoubtedly will be an altering influence, but for purposes of defining propellant-system applications, the program is well determined for approximately the next ten years. The mission criteria, as well as data regarding spacecraft weights that may be expected in view of the current booster development program, have enabled the accomplishment of detailed system analyses by JPL in all of the disciplines of significance in spacecraft operation. Thus, propulsion-system analysis can be viewed in terms of the restraints imposed by considerations such as guidance component capabilities.

Table I presents the currently anticipated program for unmanned lunar exploration. For the purposes of this discussion the significant items are: (1) every anticipated mission after the first two Ranger checkout flights includes the requirement for a post-injection trajectory correction; (2) the magnitude of the velocity increment involved in each of the post-injection maneuvers, whether the impulse is delivered in one or several increments, is only 100 to 120 ft/sec; (3) the more advanced systems involve retro-propulsion devices, and these involve velocity increments in the thousands of feet per second; (4) although not stated, there is an implicit requirement for a stabilized spacecraft, therefore necessitating some form of attitude control.

TABLE I. ANTICIPATED UNMANNED LUNAR EXPLORATION PROGRAM

Vehicle	Mission	No.	Post-injection Propulsion	Retro Propulsion	Date
Ranger Atlas-Agena B	Probe, highly elliptical orbit	2	None	None	1961
	Rough landing capsule	3	50 lb thrust N_2H_4 system ΔV = 120 ft/sec	None for main spacecraft; solid-propellant motor for split capsule	1962
Surveyor Centaur	Soft lander	7	Single start ΔV = 100 ft/sec	ΔV = 8500 ft/sec	1963-65
	Precision orbiter	3	Multiple start ΔV = 120 ft/sec	ΔV = 3000 ft/sec	1964 and 1966
Prospector Saturn	Soft lander (mobile)	10	Single start ΔV = 100 ft/sec	ΔV = 8500 ft/sec	1966-70

Table II presents the program for unmanned planetary exploration. Once again the following should be noted: (1) every mission anticipated will require the use of a post-injection propulsion system; (2) the velocity increments involved are in the realm of hundreds of feet per second (in the

TABLE II. ANTICIPATED UNMANNED PLANETARY EXPLORATION PROGRAM

Vehicle	Mission	No.	Post-injection Propulsion	Retro Propulsion	Date
Mariner-A Centaur	Venus fly-by	1	Multiple start 50 lb thrust N_2H_4 system ΔV = 250 ft/sec	None	1962
	Deep-space probe	1			
Mariner-B Centaur	Developmental shot	1	Multiple start 200 lb thrust ΔV = 500-800 ft/sec	Possible requirement	1963
	Venus fly-by	1			1964
	Mars fly-by	1			1965
	Venus fly-by	1			1966
Voyager-C Saturn	Deep-space probe	2	Multiple start 200 lb thrust ΔV = 500 ft/sec	None	1965
	Venus orbiter	2		ΔV = 7500 ft/sec	1966
	Mars orbiter	1		ΔV = 7500 ft/sec	1967
Voyager-D Saturn	Venus orbiter and/or lander	1	Multiple start 500-1000 lb thrust ΔV = 500 ft/sec	ΔV = 7500 ft/sec	1967
	Mercury orbiter and/or lander	1			1968
	Mars orbiter and/or lander	1			1969
	Venus orbiter and/or lander	1			1969
	Jupiter orbiter and/or lander	1			1970

missions involving multiple-start systems, both midcourse and approach correction requirements are included in the total increment shown; the midcourse component, however, is only of the order of 100 to 200 ft/sec); (3) the general need for retro-propulsion systems is again noted and the magnitude of the velocity increment is well into the thousands of feet per second.

Definition of Propulsion-System Characteristics. From the foregoing it is clear that a requirement for a propulsion system to accomplish post-injection maneuvers exists, that it will have wide use in virtually all space-exploration missions, and that its size is relatively small. It is of interest now to look into the details of the character of a particular propulsion system to carry out these tasks.

Analyses of the question of applying several impulse corrections in many of the projected missions has led to the interesting conclusion that liquid-propellant propulsion systems afford significant weight savings over solid-propellant propulsion units. This conclusion involves a rather complex statistical analysis, the development of which is beyond this particular discussion. The generation of the statistical analysis, however, is contained in Appendix A.

A general appreciation of the concept can be obtained by assuming first that all of the performance implications of a solid and a liquid system are identical. Next, assume that for each firing of the propulsion system, the anticipated maximum correction can be predetermined from guidance-system parameters--e.g., for the first impulse one might predict that a velocity increment of 100 ft/sec would meet 99% of all possible demands--but when the spacecraft actually flies, the velocity increment required will usually prove to be less than 100 ft/sec.

Consider first the mechanization of the solid-propellant system. In this case separate motors with sufficient propellant to carry out the maximum velocity increment for each of the separate corrections must be supplied, for although a variable total impulse can be obtained by blowing the head off the motor, for example, the propellant remaining in each case is wasted. Of course, it might be argued that a multitude of small motors could be supplied. (Consideration of system complexity, however, would seem to rule this out.) Now, in the case of the liquid-propellant propulsion system, after the first firing is accomplished there will still be some propellant left in the reservoir. This propellant is obviously not wasted but is available for use in the next firing.

The significance of this condition is brought out through statistical analysis. It can be shown that for the same percentage of over-all performance reliability for both the solid and liquid systems, the liquid system need carry only a fraction of the propellant necessary in the solid rocket. This quantity of propellant is a function of the number of firings anticipated. The ability to utilize the propellant remaining after early velocity corrections results in a 10% to 40% saving in propellant, with no loss in the surety of the mission completion.

Briefly, let us explore the restraints placed upon the propulsion devices by considerations of the guidance system. The thrust level and, consequently, the burning time selected for the post-injection propulsion applications are definitely biased by the state-of-the-art of the electronic guidance and control systems. Low thrust levels are desirable from the standpoint of minimum attitude control and mass of the thrust chamber. However, restraints imposed by the current state-of-the-art of accelerometer resolution capability and integrator drift rate are such that low accelerations and long burning times may introduce appreciable error. It appears that the sensitivity that can be expected of the present accelerometers dictates a minimum thrust level of approximately 1/15 to 1/10 Earth g; that is, for a 1000-lb spacecraft the thrust level should be approximately 100 lb. It is interesting to note that at least for the immediate future, very low thrust devices for the applications considered here are not desirable.

Let us now consider the results of a comparative design analysis concerning the masses of representative propulsion systems to accomplish typical post-injection maneuvers required in the previously described space exploration program. There are many chemical rocket systems which can accomplish post-injection impulse requirements. Within the past year a comparative analysis was made of the following five systems: (1) a monopropellant hydrazine rocket, (2) a monopropellant Cavea-B rocket, (3) a nitrogen tetroxide — hydrazine bipropellant rocket operated at the maximum impulse mixture ratio, (4) a nitrogen tetroxide — hydrazine bipropellant rocket operated at off-maximum impulse mixture ratio, and (5) a solid-propellant rocket with mechanization for impulse control.

Cryogenic propellant systems were omitted from the investigation primarily because of the attendant storage problems. Since the correction-impulse demand will be characteristically small, the specific impulse of a propellant is somewhat subordinate to considerations of inert system mass and simplicity.

In order to select systems for various missions, a preliminary design study was performed for each of the five chemical rocket systems. These systems have been compared to determine which one permits the minimum total mass of rocket for the anticipated trajectory-correction demands of the future.

The four liquid-propellant systems were evaluated at a common design point. No attempt at individual system optimization was made. A thrust chamber pressure of 150 psia was selected as well as a nozzle expansion ratio of 50:1. The liquid propellants were assumed to be stored in aluminum tanks designed for a 270-psia working pressure. The tank design incorporated a working stress level of 50, 000 psi and a minimum fabrication wall thickness of 0.030 in. Helium pressurant, used in both the monopropellant and the bipropellant systems, was assumed to be stored in titanium tanks at 3000 psia. The working stress level used for these tank designs was 110,000 psi. Each of the systems included a flexible-bag expulsion device to permit space starting under zero acceleration. For the instances where only one start was required, explosive valves were substituted for the standard pressure-actuated valves. The valves used in these preliminary designs were sized to develop pressure drops of less than 5 psi for the propellants and 1 psi for the helium. The monopropellant systems were considered to have ignition systems capable of at least three ignitions.

Schematic diagrams of the propellant systems are presented in Fig. 1 and 2: Fig. 1 illustrates the system concepts assumed for the monopropellant rockets; Fig. 2 shows those for the bipropellant rockets. The pertinent system design parameters assumed for this analysis are presented in Table III. In accomplishing this analysis a generalized procedure was developed for estimating the weight of propulsion systems; this procedure is detailed in Appendix B. Some of the general characteristics of the propellants used in the study are presented in the succeeding paragraphs.

TABLE III. MIDCOURSE-PROPULSION-SYSTEM DESIGN AND PERFORMANCE DATA

System parameter	Monopropellant hydrazine	Monopropellant Cavea B	Bipropellant N_2O_4—N_2H_4	
			Off-maximum impulse mixture ratio	Maximum impulse mixture ratio
c^*, ft/sec	4270	4880	5000	5600
$C_{P,vac}$	1.759	1.797	1.777	1.82
I_s, lb-sec/lb	233	273	276	316
L^*, in.	125	100	40	40
p_c, psia	150	150	150	150
r	-	-	0.3	1.2
$\underline{\alpha}$, deg	15	15	15	15
γ	1.28	1.24	1.28	1.23
ϵ	50:1	50:1	50:1	50:1
ρ_{comb}, lb/ft^2	63.0	93.7	67.1	73.8
Ignition	Hypergolic with N_2O_4 start slug	Hypergolic with UDMH start slug	Hypergolic	Hypergolic

Monopropellant hydrazine is assumed to be hypergolically ignited by a nitrogen tetroxide starting cartridge and is decomposed in a catalyst bed. It is then expelled through a nozzle fabricated of high-temperature alloy

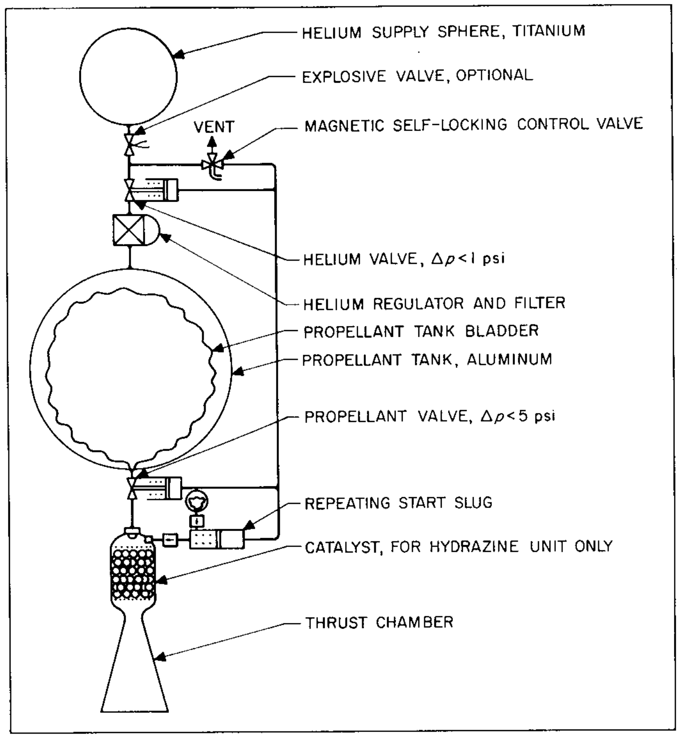

FIGURE 1. MONOPROPELLANT SYSTEM, USING HYDRAZINE OR CAVEA-B

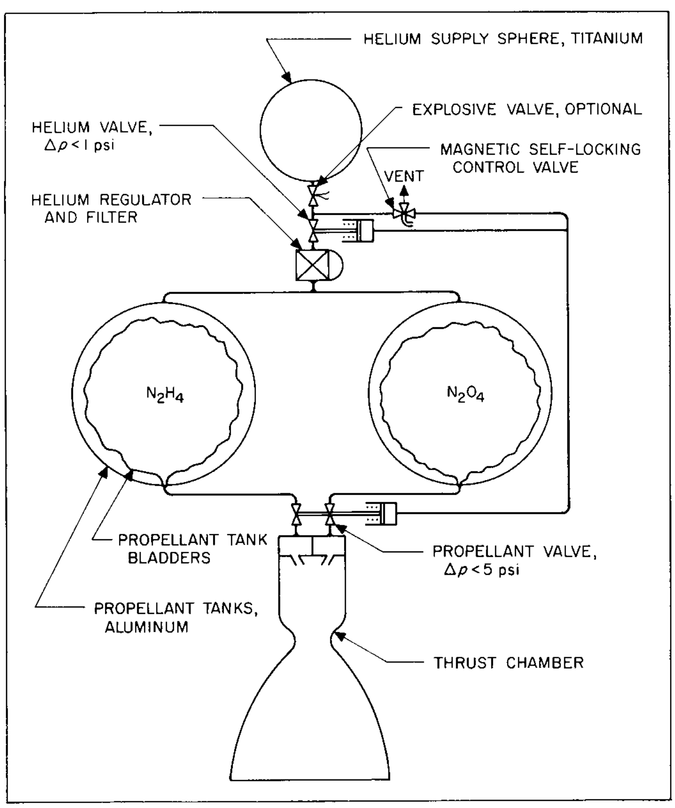

FIGURE 2. BIPROPELLANT SYSTEM, USING N_2O_4-N_2H_4

(Haynes Alloy No. 25). The effective vacuum specific impulse used was 233 lb-sec/lb.

Cavea-B is a dense, stable, viscous solution of an amine nitrate salt dissolved in white fuming nitric acid. This propellant can be hypergolically ignited with a suitable fuel such as UDMH, and has a theoretical vacuum specific impulse of approximately 294 lb-sec/lb. For this preliminary design, an impulse efficiency value of 93% was applied to account for combustion and nozzle losses. After ignition, no catalyst is required to sustain the reaction. Because of the viscous nature of this monopropellant, large valve and plumbing passages are required to minimize pressure losses during operation. Theoretical chamber temperature for the monopropellant exceeds 5000° F and, therefore, high-temperature-resistant nozzle materials are required.

The propellant combination of nitrogen tetroxide and hydrazine develops a specific impulse greater than that of a monopropellant, but adds the complexity of a bipropellant supply system. Operating at the maximum impulse mixture ratio requires use of a refractory metal or ablative thrust chamber because of the high combustion temperature. Use of off-maximum impulse mixture ratios permits employment of more conventional thrust chambers, but reduces the specific impulse delivered.

The thrust level and, consequently, burning time selection for this preliminary design study were made on the basis of anticipated guidance and control systems constraints. Considering the event of future relaxation of the accelerometer restraint, it was assumed that the burning time would never exceed 300 sec because of the guidance integrator drift constraint. In this system comparison, both limiting cases have been evaluated to illustrate the inert mass reduction possible through thrust reduction, i. e., long burning time.

A solid-propellant rocket system was also examined to determine in what applications it best fulfills the mission requirements. Design information was obtained from the performance curves presented in Ref. 1. A mass-ratio weighting factor of 0. 95 was applied to the propulsion system mass ratio to account for the necessary thrust termination components. For propellant loads less than 50 lb, a new estimate had to be made because the region was not covered in Ref. 1.

The specific impulse values utilized for the solid rocket were estimated from Fig. 3. The data upon which Fig. 3 was based--the only available data at the time of this study--were for a nozzle expansion ratio of 32 to 1. While it is recognized that this value is not totally consistent with the expansion ratios chosen for the liquid systems, the effect was felt to be minor, especially since the specific impulse was only a projected estimate.

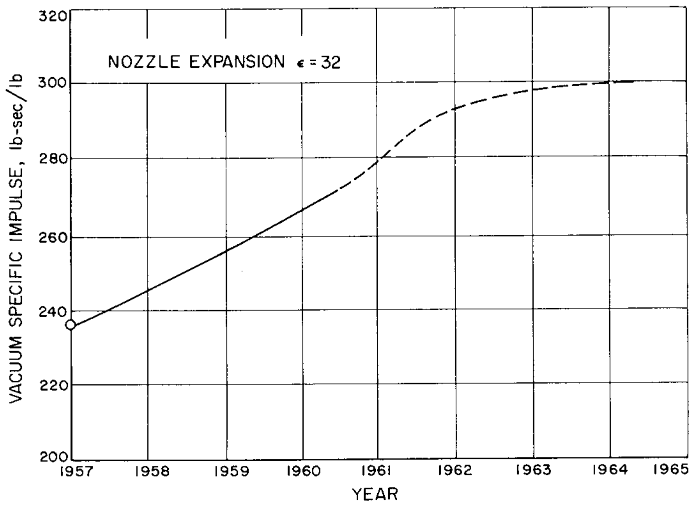

FIGURE 3. ESTIMATED VACUUM SPECIFIC IMPULSE AVAILABILITY OF SOLID-PROPELLANT ROCKETS

The results of the design analysis are presented in Table IV. Some explanation is in order concerning the term "correction geometry." The type of correction geometry--i. e., one-dimensional or two-dimensional--is determined for a particular mission by the influence coefficients of corrections in three orthogonal directions. From some injection guidance systems, a velocity correction resulting in a reduced error in only one dimension at a target interception is all that is necessary to maintain the desired accuracy. In other systems there will be a velocity correction necessary to minimize error in two dimensions. In general, the correction can be made in the plane orthogonal to the least sensitive direction of error and, therefore, a correction in three dimensions need seldom be resorted to. (See Ref. 2 for a more lucid discussion of the mechanics of computing correction requirements.) For the purpose of this study the one-dimensional and two-dimensional corrections are virtually the lower and upper limits of correction demand; the choice of one or the other for a particular mission is dependent upon vehicle parameters other than those of the propulsion system.

The results of the design analysis indicate several interesting facts. In all cases for single corrections, the solid-propellant rocket system is the least massive of the systems considered. For the missions requiring multiple corrections, it is evident that the restartable nature of the liquid-propellant systems makes even the lowest performing of them competitive

TABLE IV. COMPARISON OF MASS VALUES OF MIDCOURSE CORRECTION SYSTEMS FOR VARIOUS MISSIONS

Parameter	Centaur vehicle								
	Moon			Venus			Mars		
Estimated spacecraft mass, lb	2600	2600	2600	1950	1950	1950	1340	1340	1340
Correction geometry, dimensions	2	1	2	2	1	2	2	1	2
Number of corrections	1	2	2	1	3	3	1	3	3
Total trajectory-correction capability required, ft/sec									
Solid	100	120	120	100	500	500	100	500	500
Liquid	100	91	105	100	245	345	100	245	345
Propulsion-system mass, lb									
Solid rocket	31.8	38.6	38.6	24.0	115.3	115.3	17.0	80.3	80.3
Monopropellant N_2H_4	48.7 (66.0)[a]	46.3 (63.6)	51.8 (69.1)	38.9 (51.2)	81.8 (93.6)	109 (121)	29.6 (36.9)	59.4 (66.7)	78.4 (85.7)
Monopropellant Cavea-B	41.4 (52.7)	39.7 (50.8)	44.5 (55.8)	32.8 (42.0)	(70.0 (77.6)	93.3 (101)	24.6 (31.6)	50.1 (56.5)	66.8 (72.6)
Bipropellant $N_2O_4 - N_2H_4$, off-maximum impulse mixture ratio	43.5 (54.5)	40.9 (51.9)	45.5 (56.5)	34.6 (43.5)	71.6 (78.9)	95.1 (102)	26.1 (32.6)	52.0 (57.3)	68.2 (73.5)
Bipropellant $N_2O_4 - N_2H_4$, maximum impulse mixture ratio	39.3 (49.7)	37.5 (47.9)	41.3 (51.7)	31.4 (39.9)	64.8 (71.4)	83.6 (92.1)	23.8 (30.1)	47.6 (52.3)	61.9 (66.6)

[redacted]

TABLE IV. (Cont'd)

Parameter	Saturn C-1 vehicle					
	Moon		Venus		Mars	
Estimated spacecraft mass, lb	8000		4000		4000	
Correction geometry, dimensions	1	2	1	2	1	2
Number of corrections	2	2	3	3	3	3
Total trajectory-correction capability required, ft/sec						
Solid	120	120	500	500	500	500
Liquid	91	105	245	345	245	345
Propulsion-system mass, lb						
Solid rocket	105	105	210	210	210	210
Monopropellant N_2H_4	124 (182)[a]	138 (196)	156 (181)	212 (237)	156 (181)	212 (237)
Monopropellant Cavea-B	105 (134)	118 (147)	134 (147)	181 (194)	134 (147)	181 (194)
Bipropellant N_2O_4—N_2H_4, off-maximum impulse mixture ratio	107 (133)	120 (147)	136 (148)	182 (194)	136 (148)	182 (194)
Bipropellant N_2O_4—N_2H_4, maximum impulse mixture ratio	96 (120)	108 (132)	122 (133)	164 (175)	122 (133)	164 (175)

[redacted]

TABLE IV. (Cont'd)

Parameter	Saturn C-2 vehicle						Nova vehicle	
	Moon		Venus		Mars		Venus	
Estimated spacecraft mass, lb	14,000		10,000		10,000		100,000	
Correction geometry, dimensions	1	2	1	2	1	2	1	2
Number of corrections	2	2	3	3	3	3	3	3
Total trajectory-correction capability required, ft/sec								
Solid	120	120	500	500	500	500	500	500
Liquid	91	105	245	345	245	345	245	345
Propulsion-system mass, lb								
Solid rocket	181	181	524	524	524	524	5028	5028
Monopropellant N_2H_4	211 (309)[a]	237 (335)	379 (441)	519 (580)	379 (441)	519 (580)	3726	5125
Monopropellant Cavea-B	177 (223)	199 (245)	316 (345)	432 (461)	316 (345)	432 (461)	3027	4197
Bipropellant $N_2O_4 - N_2H_4$, off-maximum impulse mixture ratio	178 (220)	201 (242)	319 (345)	437 (463)	319 (345)	437 (463)	3047	3935
Bipropellant $N_2O_4 - N_2H_4$, maximum impulse mixture ratio	159 (196)	181 (217)	283 (307)	385 (409)	283 (307)	385 (409)	2676	3711

[a]Numbers in parentheses indicate system mass for 0.1-g acceleration limit; all others are for maximum burning time of 300 sec.

with the solid-propellant rocket system. It is interesting to realize that in all cases the total weight of any of the post-injection propulsion systems constitutes only 5% or less of the gross weight of the spacecraft. Finally, the results indicate that the influence of the guidance system constraints, such as accelerometer resolution and required correction geometry, may increase the mass of the liquid-propellant propulsion system by as much as 50%.

It would appear that the results of the foregoing studies present a good case for liquid-propellant systems. However, the choice between a monopropellant and a bipropellant system is evident only after considering all of the constraints presented by the spacecraft design. Clearly the bipropellant systems are lighter because of specific impulse alone; but for most of the missions considered, weight alone is a secondary problem. A differential of a few pounds in a 1000-lb spacecraft can easily be overlooked if the alternative is the use of a more reliable, more flexible, simpler-to-operate system.

The fact that these alternate items are important and can overshadow performance is well evidenced by the Ranger spacecraft system. The Ranger, whose ultimate mission is to rough-land a capsule on the Moon, weighs approximately 725 lb. The midcourse propulsion system is required to operate only once, yet in the over-all system analysis--when considerations of development time, ease of packaging, ability to defer decision on specific tank sizes until late in the development program were made--it was concluded that a liquid monopropellant system would be utilized.

The propulsion system for the Ranger was developed at the Jet Propulsion Laboratory and consists of a small, monopropellant hydrazine system of 50 lb vacuum thrust. The unit is capable of delivering a variable total impulse to the spacecraft by command of an integrating accelerometer system. The duration of engine operation is determined by comparison of a ground-commanded velocity increment, determined from tracking data, with a velocity increment computed by the on-board integrating accelerometer system. A chronological description of the development program is contained in Ref. 3 through 11.

Functionally, the propulsion system is a pressure-fed constant-thrust rocket. Injection pressure is derived from compressed helium gas which passes through a pressure regulator and forces the fuel from a bladdered propellant tank into the rocket engine. The rocket engine contains pelleted catalyst to accelerate and control the decomposition of the anhydrous hydrazine monopropellant. Rocket engine ignition is accomplished through the injection of a small quantity of hypergolic oxidizer, nitrogen tetroxide. All valving functions for the unit are accomplished with explosively actuated valves. A photograph of the unit is shown in Fig. 4.

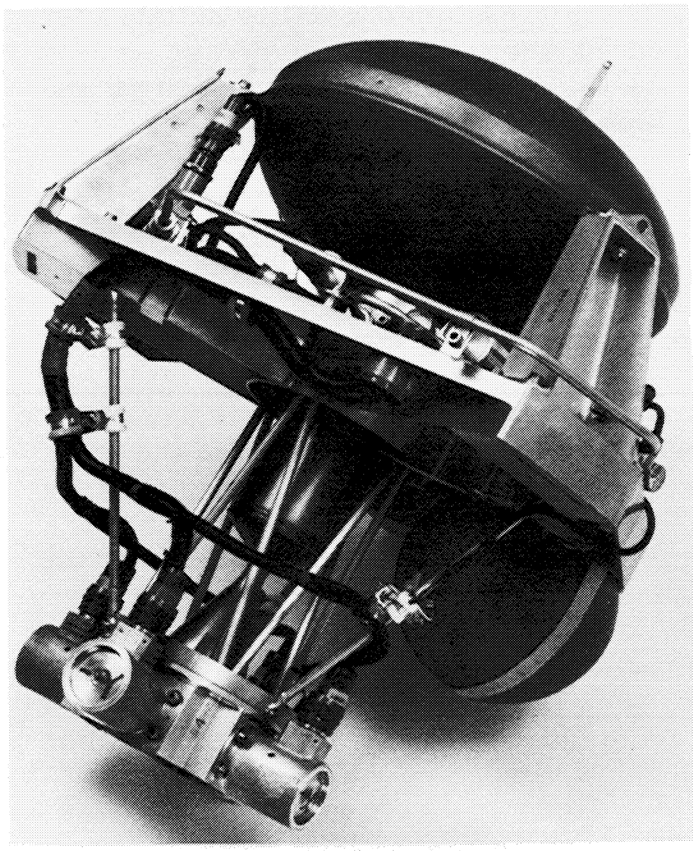

FIGURE 4. RANGER MID-COURSE PROPULSION UNIT

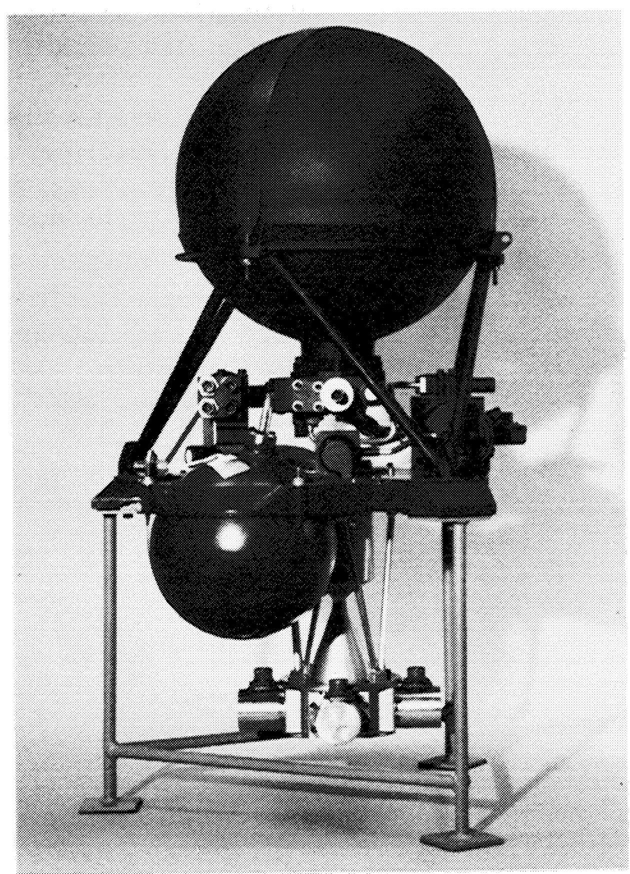

FIGURE 5. MARINER-A MID-COURSE AND APPROACH CORRECTION PROPULSION UNIT

In addition, a monopropellant hydrazine propulsion system, utilizing the same thrust chamber but incorporating a restart capability suitable for up to five ignitions, is being developed for use in the Mariner-A spacecraft system. This system is shown in Fig. 5. A description of the system and the status of its development can be found in Ref. 6 through 11.

SUMMARY AND CONCLUSIONS

The various studies which have been discussed have indicated that the following criteria govern the characteristics of a propulsion system for use as a post-injection maneuver device:

1. The unit must impart a velocity increment of from 100 to a few hundred feet per second. In most cases the unit will have to deliver this in several impulses.

2. The total system weight will be about 5%, or less, of the gross weight of the spacecraft.

3. Thrust level will be moderate because of restraints of the guidance system. For the foreseeable future, the thrust level will range from approximately 50 to a thousand pounds.

4. Missions requiring multiple corrections favor liquid-propellant systems where a common reservoir is used for the propellant.

These criteria--as well as the implicit desirability of obtaining maximum propulsion-system reliability, simplicity, and flexibility--indicate that liquid monopropellant systems possess a high order of acceptability for such applications. This is borne out by the fact that the first two midcourse propulsion systems to be developed utilize monopropellants. For monopropellants to maintian their acceptability in post-injection devices in the future, effort must be expended to develop propellants of higher energy in order that the monopropellant performance level remains competitive. But perhaps even more important, for these applications the propellants must maintain the capability of high reliability, flexibility, and simplicity in their operation.

APPENDIX A

MIDCOURSE-CORRECTION PROPELLANT UTILIZATION

In sophisticated space missions of the future it will be necessary to make several in-transit velocity corrections. The velocity increment required for each correction can be computed from the injection and guidance errors; however, a detailed analysis of these errors is beyond the scope of this study on propellant utilization. Some sources suggested for further information are Ref. 2, 12, 13, and 14.

There are at least two methods of accomplishing multiple velocity corrections: (1) use of a separate propulsion system for each correction, and (2) use of one restartable propulsion system for all corrections. The advantage of the restartable system is that the unused propellant remaining after the early velocity corrections is available for use in succeeding corrections.

In determining the in-transit correction needs for a given mission there is an option available. Either the corrections may be required in three-dimensional space to maintain both impact accuracy and desired arrival time, or corrections may be limited to two dimensions to insure only impact accuracy but not necessarily to match a predetermined arrival time. In the latter operation, corrections are made in the so-called critical plane. As would be expected, the former is more extravagant of propellant and, for most practical applications, is more sophisticated than is warranted. For comparative purposes, however, the propellant savings of the single-tank system over the multiple-tank system are determined herein for both two-and three-dimensional corrections.

For the purpose of this analysis, it was assumed that for each correction the velocity demand in each of the (two or three) mutually perpendicular directions in space is normally distributed about zero, and that each demand is statistically independent of the others; that is, correlation coefficients are zero. Also, the resultant velocity demand for each correction is assumed independent of the demand for previous or ensuing corrections.

<u>Propellant Supply for a Multiple-Reservoir System</u>. One technique for making multiple midcourse corrections is to use a separate propulsion system capable of meeting a chosen percentage of the possible correction velocity demands for each correction. In order to cover all of the corrective situations, the midcourse propulsion system would grow to prohibitive proportions. If for n corrections the velocity correction increments V_{1t}, V_{2t}, . . . , V_{nt} correspond to the P_i confidence interval for each, the probability of successful mission completion is reduced to the product of the individual probabilities of the n maneuvers:

$$P_m \equiv P_{mission} = P_1 \cdot P_2 \cdot P_3 \cdots P_n$$

By including n separate systems with a total velocity capability of $V_{1t} + V_{2t} + V_{3t} + \cdots V_{nt}$, the mission surety of completion is reduced to $(P_i)^n$ for the special case of equal surety of completion for each correction. This is the case considered for the comparisons of Table A-I (see the Nomenclature for definition of symbols).

TABLE A-I. PREDICTED PROPELLANT SAVINGS OF SINGLE-RESERVOIR MIDCOURSE CORRECTION ROCKET OVER MULTIPLE-RESERVOIR SYSTEM

P_i	n	P_m	V_{1t}	V_{2t}	V_{3t}	Predicted savings,[a] %
Two-dimensional corrections						
0.99	2	0.98	100	20	0	12 to 24
0.99	2	0.98	100	50	0	21 to 31
0.99	2	0.98	100	100	0	24 to 37
0.99	3	0.97	100	100	100	32
0.99	3	0.97	100	100	50	31
0.95	2	0.90	100	20	0	11 to 29
0.95	2	0.90	100	50	0	20 to 37
0.95	2	0.90	100	100	0	24 to 40
0.95	3	0.86	100	100	100	28
0.95	3	0.86	100	100	50	27
Three-dimensional corrections						
0.99	2	0.98	100	100	0	21
0.99	3	0.97	100	100	100	29
0.99	3	0.97	100	100	50	28
0.95	2	0.90	100	100	0	20
0.95	3	0.86	100	100	100	28
0.95	3	0.86	100	100	50	27

[a]The savings shown are independent of the magnitude of the individual correction needs; to compute the savings, it is necessary to know only the relative proportion of each of the correction velocities.

<u>Propellant Supply for a Single-Reservoir Restartable System.</u> Another technique for multiple corrections is to include a single-reservoir, restartable propulsion system with the capability of performing all the corrections with the same probability of success for the total mission as for the

multiple-unit system. The advantage of this method of correction is that the unused propellant from early corrections is available for use in succeeding ones, since the statistical chance of using the maximum capability in every correction is quite small. For the multiple-reservoir system mentioned earlier, the residuals from each correction are unavailable for further use.

The problem posed in the case of a single-reservoir system is that of determining the total velocity increment capability V_T required to assure mission completion with the same degree of confidence P_m as an appropriate multiple-tank system. Functionally this may be represented as

$$P_m = P\left[\sum_{i=1}^{n} v_{it} \le V_T\right]$$

where

$$v_{it}^2 = \sum_{j=1}^{k} v_{ij}^2$$

and k is the number of orthogonal directions in space in which corrections are to be made (two or three).

In the analysis of the velocity requirements, it is assumed that the velocity demand in each of the mutually perpendicular directions in space is normally distributed about zero and statistically independent of the demand in other directions for the same correction maneuver. Also, the resultant velocity demand magnitude v_{it} for a correction is assumed independent of all other corrections.

If for the ith correction the values of σ_{i1}^2, σ_{i2}^2, and σ_{i3}^2 correspond to the variances of the normally distributed velocity demands in each of the velocity reference directions, the total velocity capability necessary may be determined implicitly from the evaluation of the following multiple integral of the probability distribution function:

$$P_m = \int^{V_T} \int \cdots 3n \int \prod_{i=1}^{n} d\, P_i$$

$$P_m = \int^{V_T} \int \cdots 3n \int \prod_{i=1}^{n} \left\{ \frac{1}{(2\pi)^{3/2}(\sigma_{i1}\sigma_{i2}\sigma_{i3})^{1/2}} \right.$$

$$\left. \times \exp\left[-1/2 \left(\frac{v_{i1}^2}{\sigma_{i1}^2} - \frac{v_{i2}^2}{\sigma_{i2}^2} + \frac{v_{i3}^2}{\sigma_{i3}^2} \right) \right] dv_{i1} dv_{i2} dv_{i3} \right\}$$

subject to the constraints that

$$V_T \geq \sum_{i=1}^{n} v_{it}$$

$$v_{it}^2 = \sum_{j=1}^{k} v_{ij}^2$$

Qualitatively these analytic functions express the conditions that the probability of completion of all the corrections is equal to the product of the probabilities of completing each individually, and that although the correction velocity demands may be in any relative proportion, the sum of the magnitudes must not exceed a fixed value. Obviously the integral is too lengthy for convenient solution in the number of cases to be investigated. In the following sections the value of V_T is determined for some arbitrary missions. An exact solution is presented for certain dual two-dimensional corrections, and an approximate solution is presented for all other cases.

Two-Dimensional Corrections. For one correction in two dimensions, the cumulative distribution function of the velocity demand may be determined from the component velocity demand distributions to be

$$P_i = P\left[v_{it}^2 \leq V_{it}^2\right] = \int^{V_{it}} \int \frac{1}{2\pi(\sigma_{i1}\sigma_{i2})^{1/2}} \exp\left[-1/2 \left(\frac{v_{i1}^2}{\sigma_{i1}^2} + \frac{v_{i2}^2}{\sigma_{i2}^2} \right) \right] dv_{i1} dv_{i2}$$

where V_{it} is the resultant velocity magnitude for the ith correction which corresponds to the P_i fractile of the distribution. For a specific application it will be necessary to know if the velocity demand is preferential in direction; i.e., is the demand distribution different in the two directions? Since this information is determined from the guidance error analysis, it is not available at this time. In any event the limiting cases may be intestigated. It may be shown that all cases of two-dimensional correction lie in the range from $\sigma_{i1}/\sigma_{i2} = 1$ to $\sigma_{i1}/\sigma_{i2} = \infty$. The solution for these limiting cases is relatively simple, and from these the possible spread of values may be determined.

The stipulation of two-dimensional corrections does not imply that all corrections are made in the same two dimensions. It is only necessary that the effect of velocity errors in the third dimension for each correction be neglected.

<u>Case I</u>

$$\sigma_{i1} = \sigma_{i2} = \sigma_i$$

For the case of identical demand distributions in both directions, the cumulative distribution function may be represented as

$$P_i = \int^{V_{it}} \int \frac{1}{2\pi\,\sigma_i} \exp\left[-\,1/2\left(\frac{v_{i1}^2 + v_{i2}^2}{\sigma_i^2}\right)\right] dv_{i1}\, dv_{i2}$$

which in Ref. 15 is shown to be the distribution function of the familier χ^2 variable with two degrees of freedom, and may be replaced by the equivalent relation

$$P_i = \int_0^{(V_{it}/\sigma_i)^2} 1/2 \exp\left[-\,1/2\,\chi^2\right] d\chi^2$$

This is the defining relation for V_{it}, with

$$\chi_{P_i}^2 \equiv \frac{V_{it}^2}{\sigma_i^2}$$

where $\chi_{P_i}^2$ is the P_i fractile of the cumulative χ^2 (2 dt) distribution. The cumulative distribution function for dual two-dimensional correction may then be represented as

$$P_m = \int_0^{(V_T/\sigma_1)^2} \int_0^{\chi_2^2} 1/4 \exp\left[-\,1/2(\chi_1^2 + \chi_2^2)\right] d\chi_2^2\, d\chi_1^2$$

subject to the constraint that

$$\chi_2^2 = \left(\frac{V_T \sqrt{\chi_1^2}\,\sigma_1}{\sigma_2}\right)^2$$

where χ_1^2 and χ_2^2 are the random variables of the χ^2 distributions of the first and second correction demand, respectively. This function has been integrated and is presented graphically in Fig. A-1a for several values of the parameter σ_1/σ_2. It is applicable only to the case of dual two-dimensional correction.

Case II

$$\sigma_{i1} >> \sigma_{i2} \quad \sigma_{i1} \equiv \sigma_i$$

This is the case of essentially one-dimensional corrections. The demand in the other dimension is considered to be insignificant. For this case V_{it} may be defined as

$$\frac{V_{it}}{\sigma_i} \equiv u \ \frac{P_i + 1}{2}$$

where $u\left[(P_i + 1)/2\right]$ is the $(P_i + 1)/2$ fractile of the cumulative normal distribution. The P_i fractile value is not used since the absolute value (two-sided interval) of V_{it} is desired. The demand distribution for the total velocity correction requirement V_T is characterized by the parameters

$$\mu_T = 0$$

$$\sigma_T = \sqrt{\sigma_1^2 + \sigma_2^2}$$

The total velocity demand may be determined by consulting the table of fractiles of the normal distribution function, and using the following relation:

$$\frac{V_T}{\sigma_T} = u\left(\frac{P_m + 1}{2}\right)$$

The variation of total demand is presented in Fig. A-1b for several choices of σ_1/σ_2.

In order to determine the total correction velocity requirements of the single-reservoir system for missions requiring two corrections (each in two dimensions), the results of the study for the limiting cases may be applied. It is only necessary to choose a value of P_i and to know the relative magnitude of the two corrections σ_1/σ_2. This information must be available to the propulsion system designer for a specific mission. For Case I ($\sigma_{i1}/\sigma_{i2} = 1$), the ratio of the required velocity capability for the single reservoir to that for the multiple reservoir may be determined from the following:

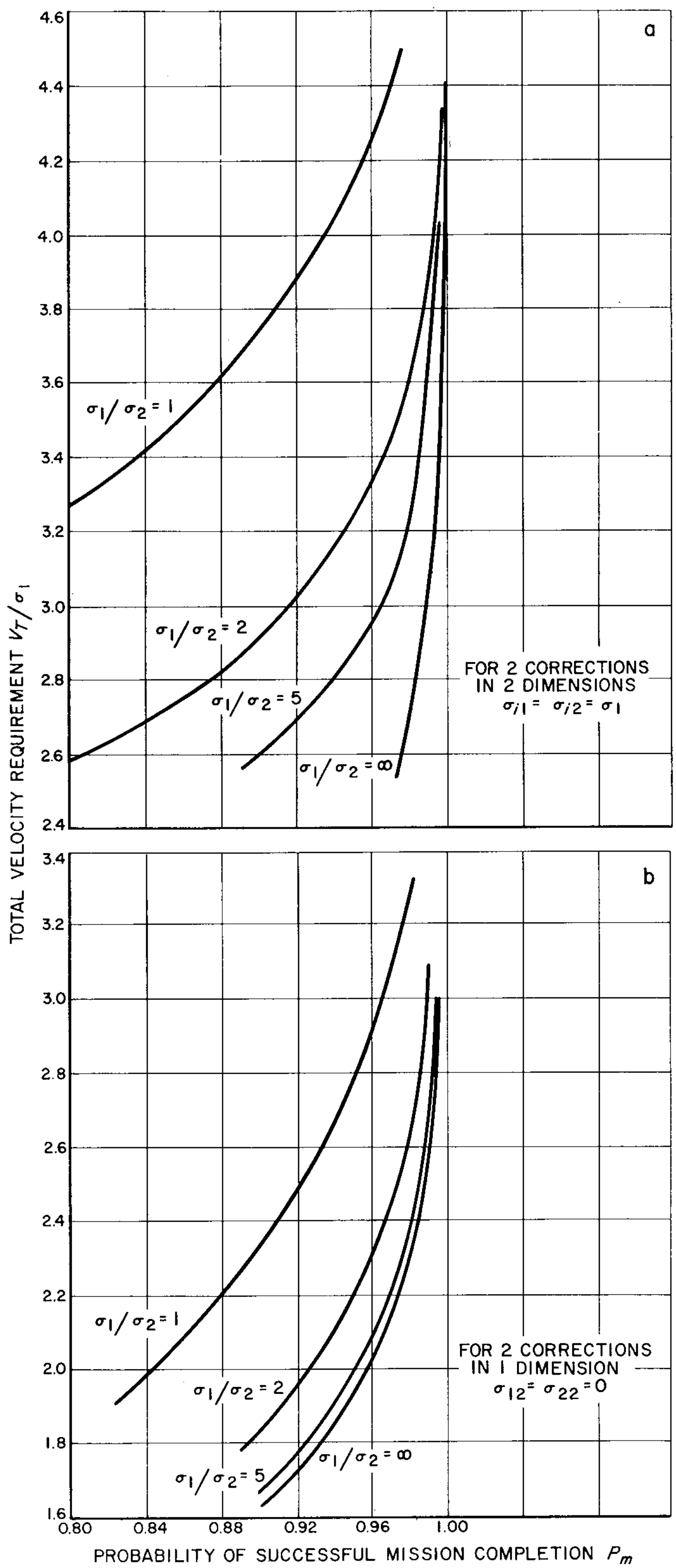

FIGURE A-1. TOTAL CORRECTION VELOCITY REQUIREMENT VS PROBABILITY OF SUCCESSFUL MISSION

$$\frac{V_T}{V_{1t} + V_{2t}} = \frac{V_T/\sigma_1(P_i;\ \sigma_2/\sigma_1)}{(1 + \sigma_2/\sigma_1)\ \sqrt{\chi^2_{P_i}}}$$

where V_T/σ_1 is obtained from Fig. A-1a and $\chi^2_{P_i}$ is obtained from the tables of χ^2.

For Case II ($\sigma_{i1}/\sigma_{i2} = \infty$), this ratio may be shown to be

$$\frac{V_T}{V_{1t} + V_{2t}} = \frac{V_T/\sigma_i(P_i;\ \sigma_2/\sigma_1)}{(1 + \sigma_2/\sigma_1)\ u\left[(P_i + 1)/2\right]}$$

where $u\left[(P_i + 1)/2\right]$ is obtained from the normal distribution function tabulation and V_T/σ_1 is obtained from Fig. A-1b. If the velocity demand distributions in the two reference directions for each correction are known, a more specific value of velocity may be determined by a numerical solution.

Three-Dimensional Corrections. The cumulative distribution function for demand on three-dimensional corrections may be represented as

$$P_i = \int\int^{V_{it}}\int \frac{1}{(2\pi)^{3/2}(\sigma_{i1}\sigma_{i2}\sigma_{i3})^{1/2}}$$

$$\times \exp\left[-1/2\left(\frac{v^2_{i1}}{\sigma^2_{i1}} + \frac{v^2_{i2}}{\sigma^2_{i2}} + \frac{v^2_{i3}}{\sigma^2_{i3}}\right)\right] dv_{i1}dv_{i2}dv_{i3}$$

For the case of identical distributions in the three dimensions, the distribution of v_{it}/σ_i is identical to that of a χ^2 random variable with three degrees of freedom (Ref. 12). The above expression defines V_{it}, and for the case of identical component velocity distributions $\sigma_{i1} = \sigma_{i2} = \sigma_{i3} = \sigma_i$

$$\chi^2_{P_i} \equiv V^2_{it}/\sigma^2_i$$

where $\chi^2_{P_i}$ is the P_i fractile of the χ^2 (three degrees of freedom) distribution.

The integral for several three-dimensional corrections is extremely complicated and will not be attempted. For all cases of three-dimensional corrections and for those two-dimensional corrections not treated previously, an approximation technique has been applied.

It is necessary to determine the distribution of the sum of the magnitudes of the n-velocity correction demands in order to compute the total velocity capability needed to cover the desired percentage of possible situations. The square of the correction velocity demand (v_{it}^2/σ_i^2) for each correction has been shown to be distributed as a χ^2 random variable, if the normal demand distributions in the (two or three) orthogonal directions are identical. Fortunately, the $\sqrt{\chi^2}$ is approximately normally distributed for two and three degrees of freedom. (The approximation is better for the case of three degrees of freedom than for two degrees of freedom.) The magnitude of the quantity v_{it}/σ_i is, therefore, approximately normally distributed, and the nondimensional total velocity demand distribution

$$\sum_{i=1}^{n} v_{it}/\sigma_i$$

is merely the distribution of the sum of n nearly normally distribution populations.

For purposes of this first analysis, the normal approximations were determined by visually choosing the normal distribution to best fit the $\sqrt{\chi^2} \equiv v_{it}/\sigma_i$ curve. The best fit distributions for v_{it}/σ_i were found to be described by the following:

(1) For two dimensions (two degrees of freedom),

$$\mu = 1.34$$

$$\sigma = 0.67$$

(2) For three dimensions (three degrees of freedom),

$$\mu = 1.54$$

$$\sigma = 0.77$$

Then, using the rule of addition for normal populations, the distribution of the sum of n demands may be represented as a normal distribution with the following parameters:

(1) For two dimensions,

$$\mu_T = 1.34 \sum_{i=1}^{n} \sigma_i$$

$$\sigma_T^2 = (0.67)^2 \sum_{i=1}^{n} \sigma_i^2$$

(2) For three dimensions,

$$\mu_T = 1.54 \sum_{i=1}^{n} \sigma_i$$

$$\sigma_T^2 = (0.77)^2 \sum_{i=1}^{n} \sigma_i^2$$

Therefore, the total velocity capability for the n corrections with the single-tank system may be approximated by consulting the tables of the normal distribution function, given the cumulative probability of mission completion P_m; that is,

$$\frac{V_T - \mu_T}{\sigma_T} = u_{P_m}$$

where u_{P_m} is the P_m fractile of the standard normal distribution and is obtained from the tables.

Calculation of the Savings. The velocity capability necessary to complete the mission with the same degree of surety as the multiple-tank system is V_T. The propellant savings of the single-tank system is reflected in the decreased velocity capability necessary for equivalent coverage. The percentage savings may be represented as propellant savings:
$\% = 100\left\{1 - \left[V_t/(V_{1t} + V_{2t} + \cdot\cdot\cdot\ V_{nt})\right]\right\}$.

Results. The propellant savings of the single-tank system over the multiple-tank system were computed by the methods discussed and are presented in Table A-I. The examples were chosen to cover the expected range of interest; however, the exact number and relative magnitude of the corrections necessary for specific space missions are not known at this time.

For the dual two-dimensional corrections in Table A-I, the two values of propellant savings presented are for the limiting cases: (1) when the velocity demand distributions in both directions are identical for each correction, and (2) the degenerate case, when the velocity demand distributions are essentially one-dimensional. Any other relationship between the normal distributions of velocity demand in the two directions will yield propellant savings somewhere between the limits shown.

The savings shown for the other cases represent the application of an approximate solution for the instance of identical velocity demand distribution in all directions for each correction. An exact solution is possible, but is too lengthy for convenient evaluation in the number of cases presented.

The single valued savings in Table A-I represent the lower limit of savings for the missions analyzed.

APPENDIX B

DETERMINATION OF PROPULSION SYSTEM MASS

A generalized procedure has been developed which allows the determination of the mass of any post-injection maneuver system using the previously described propellants. Given the spacecraft mass and the velocity to be gained--i.e., the correction impulse--the propellant mass that must be expended may be determined from the familiar rocket equation

$$M_p = M_g \left[1 - \exp\left(- \frac{\Delta V}{g I_s}\right)\right] \tag{B-1}$$

For the liquid-propellant systems, the specific impulse may be obtained from Table III. The system mass per unit of propellant mass (M_{sp}/M_p) may be obtained from Fig. B-1, B-2, B-3, or B-4, depending upon the choice of propellant. The total system mass proportional to propellant mass is the product of the propellant mass and system mass per unit propellant mass:

$$M_{sp} = M_p(M_{sp}/M_p) \tag{B-2}$$

If the thrust level is not chosen, but the maximum allowable burning time is fixed, the appropriate thrust level may be computed from the following:

$$(F)_{min} = I_s M_p / t_{b,\ max} \tag{B-3}$$

For acceleration-limited systems, the thrust level selection is obvious. Using the appropriate thrust level, the system mass proportional to thrust may be determined from Fig. B-5, B-6, B-7, or B-8, for the desired propulsion system, in the same manner as the propellant-dependent system mass was found. The total correction-system mass is the sum of the propellant mass, the system mass dependent upon propellant, and the system mass dependent upon thrust.

For the solid-propellant system designs, the propellant mass may be computed from Eq. (B-1). The specific impulse predicted to be available for a given flight date is given in Fig. 3. The system mass for a solid rocket of known propellant load may be computed from the information of Fig. B-9 and is also a function of launch date. These curves are excerpted from Ref. 1. The mass of a solid rocket system is, within limits, independent of thrust level. Therefore, there is no system mass addition dependent upon thrust level.

The total correction-system mass may then be determined from Fig. B-9 to be the quotient of propellant mass and propulsion system mass

[redacted]

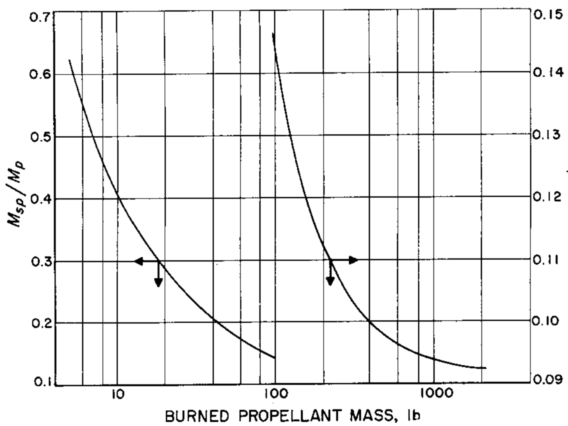

FIGURE B-1. PROPELLANT-SCALED SYSTEM MASS AS A FUNCTION OF BURNED PROPELLANT MASS FOR A HYDRAZINE MONOPROPELLANT ROCKET

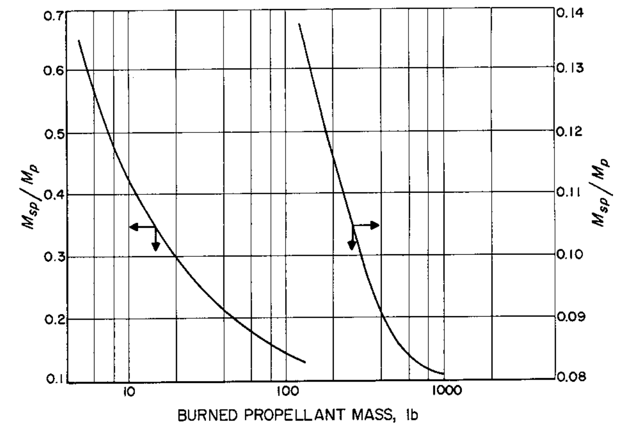

FIGURE B-2. PROPELLANT-SCALED SYSTEM MASS AS A FUNCTION OF BURNED PROPELLANT MASS FOR A MAXIMUM IMPULSE N_2O_4-N_2H_4 ROCKET

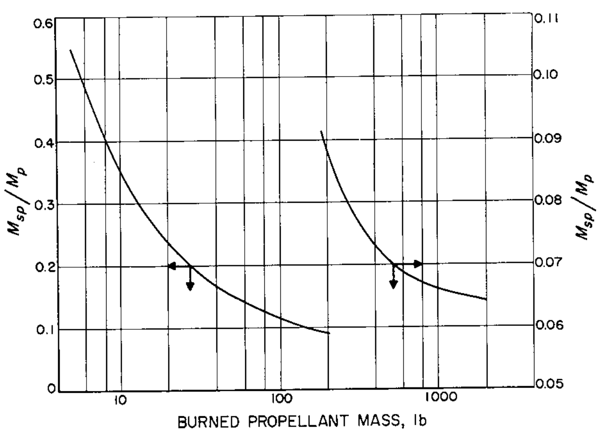

FIGURE B-3. PROPELLANT-SCALED SYSTEM MASS AS A FUNCTION OF BURNED PROPELLANT MASS FOR A CAVEA-B MONOPROPELLANT ROCKET

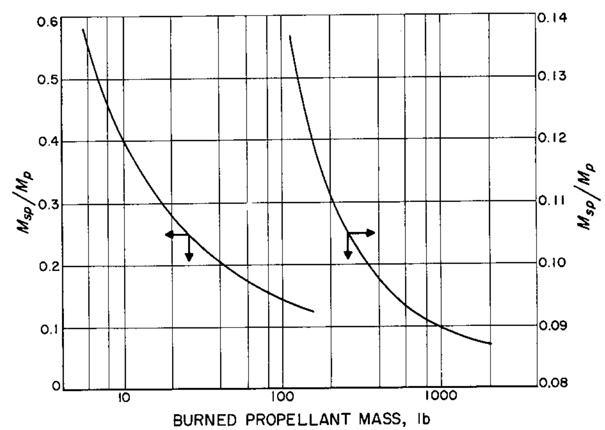

FIGURE B-4. PROPELLANT-SCALED SYSTEM MASS AS A FUNCTION OF BURNED PROPELLANT MASS FOR AN OFF-MAXIMUM IMPULSE N_2O_4-N_2H_4 ROCKET

[redacted]

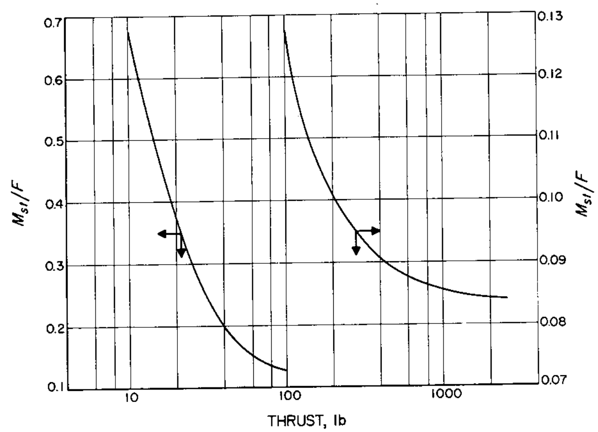

FIGURE B-5. THRUST-SCALED SYSTEM MASS AS A FUNCTION OF THRUST LEVEL FOR A HYDRAZINE MONOPROPELLANT ROCKET

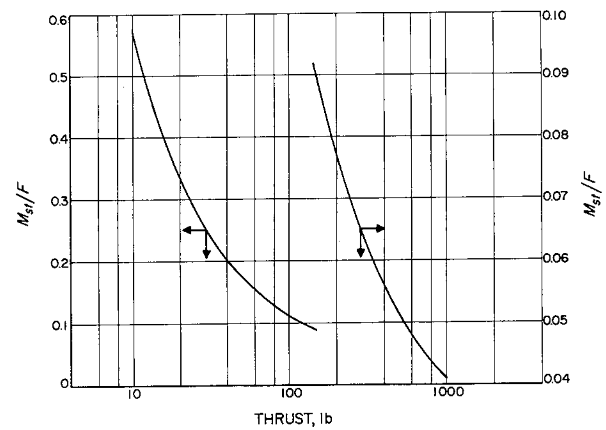

FIGURE B-6. THRUST-SCALED SYSTEM MASS AS A FUNCTION OF THRUST LEVEL FOR A MAXIMUM IMPULSE N_2O_4-N_2H_4 ROCKET

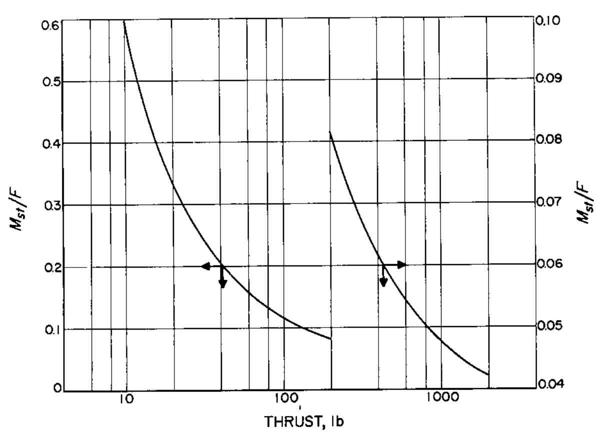

FIGURE B-7. THRUST-SCALED SYSTEM MASS AS A FUNCTION OF THRUST LEVEL FOR A CAVEA-B MONOPROPELLANT ROCKET

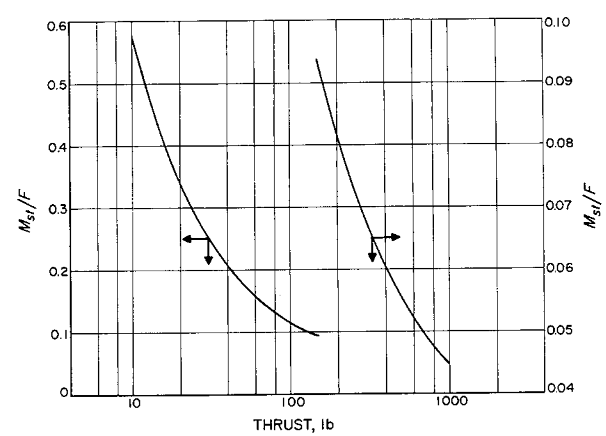

FIGURE B-8. THRUST-SCALED SYSTEM MASS AS A FUNCTION OF THRUST LEVEL FOR AN OFF-MAXIMUM IMPULSE N_2O_4-N_2H_4 ROCKET fraction. Thrust termination capability has been included in the solid rocket designs. For multiple corrections, the effect of staging the used motor cases may be included. In general, this will be of negligible consequence.

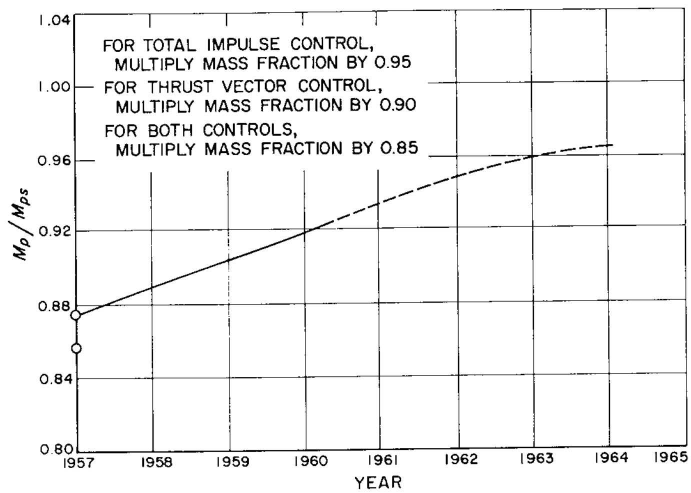

FIGURE B-9. ESTIMATED MASS FRACTION AVAILABILITY OF SOLID-PROPELLANT ROCKETS

NOMENCLATURE

F = vacuum thrust

F_{min} = minimum thrust

g = gravitational constant

I_s = vacuum specific impulse

M_g = gross injected spacecraft mass

M_p = total propellant mass

M_{ps} = total propulsion system mass, including propellant

M_{sp} = propellant-scaled mass

NOMENCLATURE (Cont'd)

M_{st} = thrust-scaled mass

n = number of individual corrections to be made in flight

P_i = prescribed probability of successful completion of ith correction maneuver

P_m = probability of successful completion of total mission velocity correction demand

Δp = valve pressure drop

$t_{b,\ max}$ = maximum burning time

u_P = value of normally distributed random variable corresponding to P fractile

v_{ij} = random variable, vehicle velocity correction demand for ith correction in jth orthogonal direction

v_{it} = random variable, resultant velocity correction demand magnitude for ith correction

V_{it} = magnitude of random variable of resultant velocity correction for ith correction which corresponds to P_i confidence interval

V_T = total velocity correction capability of single tank system

ΔV = vacuum velocity increment capability required

ΔV_{max} = maximum velocity increment for any individual correction

ϵ = expansion ratio

μ = mean value of normal distribution

σ^2 = variance of normal distribution

σ_i^2 = prescribed variance of normal distribution in each orthogonal direction for ith correction in case of identical distributions

σ_{ij}^2 = prescribed variance of normal distribution of velocity demand for ith correction and jth orthogonal direction

χ^2 = random variable of χ^2 distribution

NOMENCLATURE (Cont'd)

χ_P^2 = value of the random variable χ^2 corresponding to P fractile of cumulative distribution function.

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